A Study of Vorticity of MHD Visco-elastic Boundary layer flow through porous medium with free convection past a continuous moving surface

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ABSTRACT

This paper investigates the study of vorticity of MHD viscoelastic boundary layer flow through porous medium with free convection past a continuous surface. The effects of the important flow parameters such as Magnetic Parameter (M), Grashof number (G_r), Modified Grashof number (G_m), Prandtl number (P_r) and Schimdt number (S_c) on the vorticity of the flow field are analyzed quantitatively with the help of figures and tables.

KEYWORDS MHD flow, Visco-elastic, Porous medium, Moving surface, Vorticity.

NOMENCLATURE

MHD	=	Megneto hydrodynamics
u, v	=	Velocity Components along the (x, y) directions respectively
M	=	Magnetic Parameter
B_0	=	Magnetic Induction
κ^*	=	PermeabilityParameter
g	=	Acceleration due to gravity
v_0	=	Suction velocity
q	=	Heat flux
T	=	Fluid Temperature
u_w	=	Issuing velocity of the fluid from the slot
C	=	Concentration of the fluid in the free stream
D	=	Mass diffusion coefficient
K	=	Thermal conductivity of the fluid

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GREEK SYMBOLS

ξ = Vorticity

ν = Kinematic Viscosity

ρ = Density

σ = Electrical Conductivity

β = Coefficient of Volume Expansion

λ_0^* = Visco - elastic Parameter

μ = Coefficient of Viscosity

α = Fluid thermal diffusivity

SUPERSCRIPTS

$/$ = Differentiation with respect to x and y respectively

$*$ = Dimensional properties

SUBSCRIPT

p = Plate

w = Wall condition

∞ = Free stream condition

INTRODUCTION

The convection problem in a porous medium has important application in geothermal reservoirs and geothermal energy extractions. It is obvious that in order to utilize the geothermal energy to maximum, one should have a complete and precise knowledge of the amount of perturbations needed to generate convection current is geothermal fluids. Also, the knowledge of the quantity of perturbations essential to entreat convection currents is minerals. Cheng and Lan
[3] and Cheng and Teckchandani [4] obtained numerical solution for the convective flow in a porous medium bounded by two isothermal parallel plates in a presence of withdrawal of the fluid. All the above mentioned studies treat the permeability and the conductivity or thermal resistance of the medium as constant or neglect the effect of porosity. Saxena *et al*. [16] studied the effect of permeability of medium of unsteady MHD flow of viscoelastic oldroyd fluid under time varying body force through a rectangular channel. Johari, R. and Singh, D. [8] studied MHD free convective flow of heat and mass transfer past a vertical porous plate. Singh *et al*.
[17] discussed the effect of permeability on MHD flow of continuously moving vertical surface with uniform heat and mass flux.

Bhagwat, B. and Kuldeep [2] have discussed MHD free convective mass transfer of a viscoelastic (Water model-B) dusty gas through a porous medium with heat source. Ogulu and Makinde [11] analyzed the unsteady hydromagnetic free convection flow of a dissipative fluid past a vertical plate with constant heat flux. Sreekanth *et al.* [18] solved the hydro-magnetic unsteady Hele-Shaw flow of a Rivlin-Erickson fluid through porous media. Dubey and Bhattacharya [6] studied the fluctuating flow of a visco-elastic fluid past an infinite flat plate with uniform suction. Ravi Kant [14] discussed the fluctuating flow of visco-elastic fluid through an infinite flat plate with variable suction in boundary layer region. Johari [9] has paid his attention towards the oscillating flow an elastico-viscous fluid past an infinite flat plate with variable suction. Puri and Kythe [13] studied the unsteady free convective visco-elastic flow past a flat plate in a rotating medium. Helmy [7] obtained the exact solution of an oscillating flow of an incompressible elastico-viscous conducting fluid with variable suction in presence of a constant magnetic field applied perpendicular to the moving porous plate. Puri [12] gives the contribution towards the unsteady flow of an elastico-viscous fluid past an infinite plate with variable suction. Mittal *et al.* [10] discussed the vorticity of fluctuating flow of a visco-elastic fluid past an infinite plate with variable suction in slip flow regime. Soundalgekar *et al*. [19] found an exact solution for MHD free convection flow past an oscillating plate. Raju and Verma [15] consider unsteady MHD free convection oscillating coquette flow through a porous medium. Cookey *et al*. [5] studied influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Alagoa *et al*. [1] discussed radiative and free convective effects of a MHD flow through a porous medium. Usman *et al*. [20] have studied the unsteady free convection and mass transfer flow of micropolar fluid embedded in a porous media.

Our objective in this communication is to study the vorticity of MHD visco-elastic boundary layer flow through porous medium with free convection past a continuous moving surface. To visualize the physical behavior of the problem, numerical solution for effect of Magnetic parameter (M), Grashof number(G_r), Prandtl number(P_r), Modified Grashof number(G_m) and Schmidt number (S_c) on vorticity have been shown graphically and interpreted thereof.

SOLUTION OF THE PROBLEM

Let us consider the boundary layer flow of an electrically conducting incompressible, visco-elastic fluid over a continuous moving flat surface through porous medium with B_0 as imposed uniform magnetic field perpendicular to the surface. Let us denote velocity components u, v in direction x and y respectively temperature and concentration denoted by T and C under these assumptions the physical variables are function of y only.

Equation of Continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \ldots(1.1)$$

Equation of linear momentum:

$$u\frac{\partial u}{\partial x}+ v\frac{\partial v}{\partial y}=g\beta\left(T-T_{\infty}\right)+g\beta^{*}\left(C-C_{\infty}\right)+\nu\frac{\partial^{2}u}{\partial y^{2}}-\lambda_{0}^{*}\left[u\frac{\partial^{3}u}{\partial x\partial y^{2}}+v\frac{\partial^{2}u}{\partial y^{2}}+\frac{\partial u}{\partial x}\frac{\partial^{2}u}{\partial y^{2}}\right]-\frac{\sigma B_{0}^{2}}{\rho}u-\frac{\nu}{\kappa^{*}}u \qquad \ldots(1.2)$$

Equation of energy:

$$u\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_{p}}\frac{\partial^{2}T}{\partial y^{2}} \qquad \ldots(1.3)$$

Equation of Concentration:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^{2}C}{\partial y^{2}} \qquad \ldots(1.4)$$

The initial boundary conditions are:

$$\left.\begin{array}{l} u=u_{w},\ v=-v_{0}=\text{constant},\ T=T_{w},\ C=C_{w} \quad at\ \ y=0 \\ u\rightarrow 0,\ T\rightarrow T_{\infty},\ C\rightarrow C_{\infty} \qquad as \quad y\rightarrow\infty \end{array}\right\} \qquad \ldots(1.5)$$

Solving equations (1.1) to (1.4) under the boundary conditions (1.5) can be written as,

$$-v_0 \frac{dv}{dy} = g\beta\left(T - T_\infty\right) + g\beta^*\left(C - C_\infty\right) + \nu \frac{d^2u}{dy^2} + \lambda_0^* \frac{d^3u}{dy^3} - \frac{\sigma B_0^2}{\rho} u - \frac{\mu}{\kappa^*} u \qquad \ldots(1.6)$$

$$-v_0 \frac{dT}{dy} = \frac{\kappa}{\rho C_p} \frac{d^2T}{dy^2} \qquad \ldots(1.7)$$

$$-v_0 \frac{dC}{dy} = D \frac{d^2C}{dy^2} \qquad \ldots(1.8)$$

The boundary conditions for the velocity, temperature and concentration field are :

$$\left.\begin{array}{l} u = u_w \ , T = T_w \ , C = C_w \quad at \ \ y = 0 \\ u \to 0 \ , T \to T_\infty \ , C \to C_\infty \ as \ \ y \to \infty \end{array}\right\}$$

Introducing the following non-dimensional quantities :

$$u = \frac{U}{u_w} \ , \ y = \frac{Y}{L} \ , \ \theta = \frac{T - T_\infty}{T_w - T_\infty} \ , \ \phi = \frac{C - C_\infty}{C_w - C_\infty},$$

$$P_r = \frac{\nu}{\alpha} \ , \ S_c = \frac{\nu}{D} \ , \ \lambda_0 = \frac{\lambda_0^*}{L^2} \ , \ M = \frac{\sigma B_0^2 L}{\rho v_0},$$

$$G_r = \frac{g\beta\left(T_w - T_\infty\right)}{\rho v_0} \quad , \quad G_m = \frac{g\beta^*\left(C_w - C_\infty\right)}{v_0 u_w}$$

Now , equations (1.6) , (1.7) and (1.8) in non-dimensional form is :

$$\lambda_0 \frac{d^3u}{dy^3} + \frac{1}{R} \frac{d^2u}{dy^2} + \frac{du}{dy} - M_1 u = -G_r \theta - G_m \phi \qquad \ldots(1.9)$$

$$\frac{d^2\theta}{dy^2} + R \, P_r \frac{d\theta}{dy} = 0 \qquad \ldots(2.0)$$

$$\frac{d^2\phi}{dy^2} + \mathrm{R}\,\mathrm{S_c}\frac{d\phi}{dy} = 0 \qquad \ldots(2.1)$$

The corresponding initial and boundary conditions in non-dimensional form are:

$$\left.\begin{array}{ll} u = 1 & at\ \ y = 0 \\ u \to 0 & as\ \ y \to \infty \end{array}\right\} \qquad \ldots(2.2)$$

Solving equation (1.9) with boundary condition (2.2), we get,

$$u = e^{-my} + \mathrm{G_r} a_1 e^{-\mathrm{RP_r} y} - \mathrm{G_r} a_1 e^{-my} + \mathrm{G_m} a_2 e^{-\mathrm{RS_c} y} - \mathrm{G_m} a_2 e^{-my} \qquad \ldots(2.3)$$

The Vorticity of the flow with suction will be:

$$\xi = -me^{-my} - \mathrm{G_r P_r R} a_1 e^{-\mathrm{RP_r} y} + \mathrm{G_r} a_1 m e^{-my} - \mathrm{G_m S_c R} a_2 e^{-\mathrm{RS_c} y} + \mathrm{G_m} a_2 m e^{-my} \qquad \ldots(2.4)$$

Where,

$$m = \frac{1}{2}\left[1 + \sqrt{1 + 4\mathrm{M}}\right],\ a_1 = -\frac{\mathrm{G_r}}{\mathrm{P_r}\left(\mathrm{P_r^2} - \mathrm{P_r} - \mathrm{M}\right)},\ a_2 = -\frac{\mathrm{G_m}}{\mathrm{S_c}\left(\mathrm{S_c^2} - \mathrm{S_c} - \mathrm{M}\right)}$$

NUMERICAL RESULTS AND CONCLUSIONS:

Table I: Values of vorticity at $\mathrm{R_e} = 0.2, \mathrm{G_m} = 2, \mathrm{S_c} = 0.6,\ \mathrm{P_r} = 0.71$ and different values of M and $\mathrm{G_r}$.

$y \to$	0	1	2	3	4	5
$\mathrm{M} = 0, \mathrm{G_r} = 2$	-1.9680	-0.9355	-0.4943	-0.2825	-0.1650	-0.0904
$\mathrm{M} = 2, \mathrm{G_r} = 4,$	14.3621	1.1498	-0.5280	-0.6600	-0.5958	-0.5165
$\mathrm{M} = 4, \mathrm{G_r} = 6$	25.2237	0.7465	-0.9809	-0.9741	-0.8523	-0.7380
$\mathrm{M} = 6, \mathrm{G_r} = 8$	37.5723	0.2911	-1.3538	-1.2528	-1.0892	-0.9437

Table II: Values of vorticity at $R_e = 0.2, G_r = 2$, $P_r = 0.71, S_c = 0.6$ and different values of M and G_m.

$y \rightarrow$	0	1	2	3	4	5
$M = 0, G_m = 2$	-1.9680	-0.9355	-0.4943	-0.2825	-0.1650	-0.0904
$M = 2, G_m = 4$	-7.7309	-0.2381	0.6896	0.7380	0.6755	-0.6053
$M = 4, G_m = 6$	-11.7734	0.3159	1.1135	1.0544	0.9425	0.8384
$M = 6, G_m = 8$	-15.4482	0.8426	1.4735	1.3448	1.1961	1.0624

Table III: Values of vorticity at $R_e = 0.2, G_r = 2$, $P_r = 0.71, S_c = 0.6$, G_m=2.0 and different values of M and S_c.

$y \rightarrow$	0	1	2	3	4	5
$M = 0, S_c = 0.6$	-1.9680	-0.9355	-0.4943	-0.2825	-0.1650	-0.0904
$M = 2, S_c = 1.2$	1.3059	0.2072	0.0305	-0.0126	-0.0311	-0.0413
$M = 4, S_c = 1.8$	0.1239	0.0531	0.0089	-0.0181	-0.0337	-0.0419
$M = 6, S_c = 2.4$	-0.7338	0.0305	0.0167	-0.0125	-0.0286	-0.0359

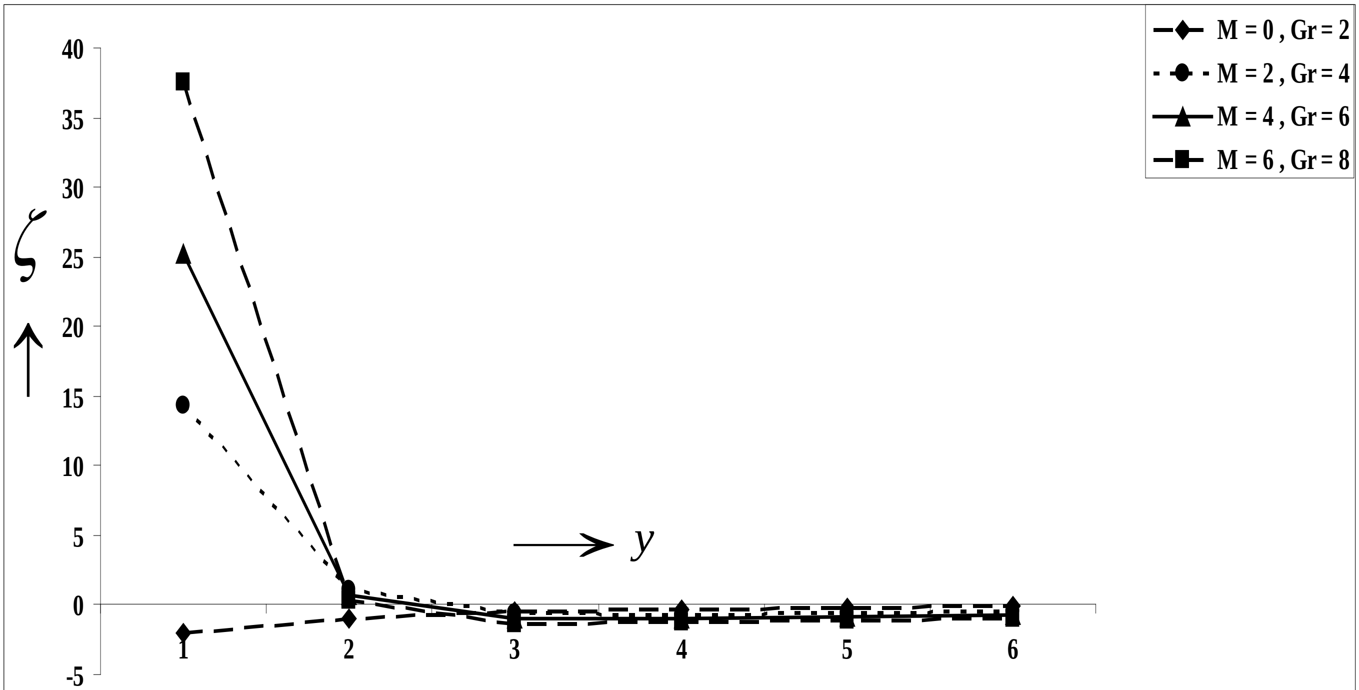

Fig.1 Vorticity profile for different values of M and Gr

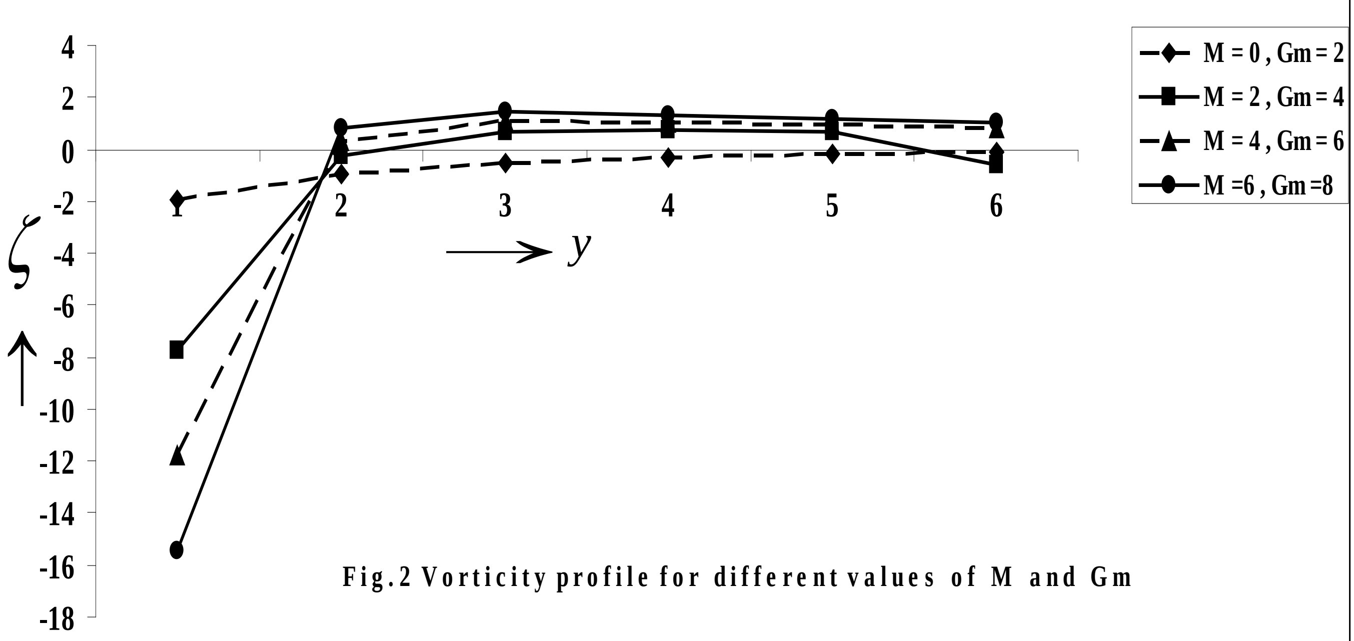

Fig.2 Vorticity profile for different values of M and Gm

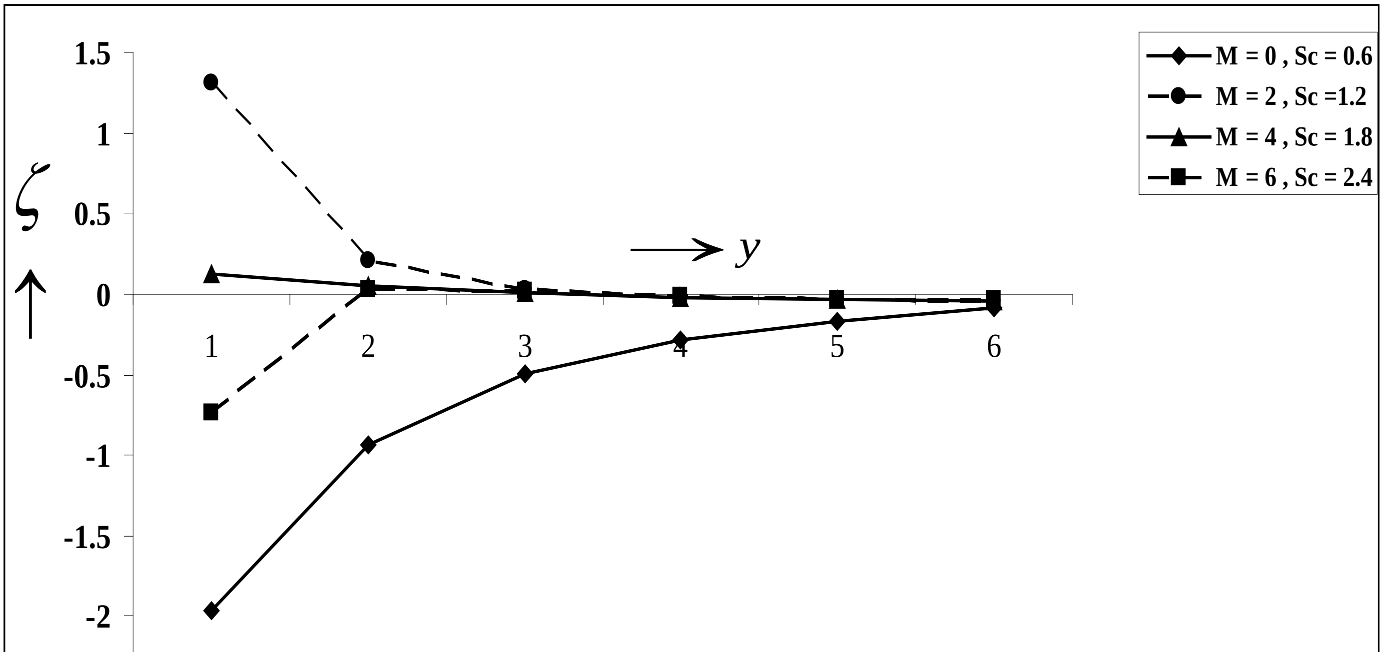

Fig.3 Vorticity profile for different values of M and Sc

In The order to understand the physical solution, we have calculated the numerical values of the vorticity distribution in figures (1) to (3) and table (I) to (III) for the different values of Magnetic parameter (M), Grashof number (G_r), Modified Grashof number (G_m), and Schmidt number (S_c).

From table (I) and figure (1): The vorticity distribution of boundary layer flow plotted against y for R_e=0.2, G_m= 2.0, S_c=0.6, P_r=0.71 and different values of magnetic parameter (M) and Grashof number (G_r).

1. For the value of Magnetic parameter (M = 0) and Grasof number ($G_r = 2$), the vorticity increases continuously from y=0 to y=5.
2. Increases the value of Magnetic parameter (M) and Grashof number (G_r) then vorticity decreases for y=0 to y=5.

From table (II) and figure (2): The vorticity distribution of boundary layer flow plotted against R_e=0.2, G_r = 2.0, S_c=0.6, P_r=0.71 and different values of magnetic parameter (M) and modified Grashof number (G_m).

1. For the value of the magnetic parameter M = 0 and modified Grashof number (G_m), the vorticity increases continuously from y = 0 to y = 5.

2. As thebvalue of M and (G_m) increases the vorticity decreases continuously at y = 0.

From table (III) and figure (3): The vorticity distribution of boundary layer flow plotted against R_e=0.2, G_r = 2.0, G_r=2, P_r=0.71 and different values of magnetic parameter (M) and Schmidt number (S_c).

1. At M = 0 and S_c = 0.6 there is increases there is increase the value of vorticity witty increase y.
2. As the value of magnetic parameter (M=2) and Schmidt number $(S_c = 1.2)$ increases the vorticity increases continuously.
3. Also for (M = 4, 6) and $(S_c = 1.8, 2.4)$ the vorticity decreases continuously.

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